For a rigid body to be in equilibrium, two vector equations must be satisfied:

$$\mathbf{F} = 0$$

$$\tau = 0$$

The first of these is the condition that the net force is zero and ensures that there is no change in the translational motion. The second is the condition that the net torque about some convenient point is zero; it ensures that the rotational motion remains the same.

For objects at rest or in motion, the entire weight and mass of the object can be considered to be located at a point called the center of gravity or center of mass. This fact facilitates the understanding of stability and balance.

The mechanical advantage (M.A.) of a lever or other simple machine is defined as the ratio F_L/F_a of the load force $\mathbf{F}_L$ balanced by an applied force $\mathbf{F}_a$. In the bodies of animals, limbs with small mechanical advantages are suited to fast motion, while those with large mechanical advantages are able to exert large forces.

Checklist

Define or explain:

rigid body	center of gravity
torque	center of mass
vector product or cross product	base area
couple	machines
equilibrium conditions	mechanical advantage

REVIEW QUESTIONS

Q4-1 A rigid body does not change its ______ or ______ when subjected to a force.

Q4-2 The quantity that indicates the ability of a force to cause rotation is called the ______.

Q4-3 The perpendicular distance from the axis of rotation to the line of action of a force is the ______.

Q4-4 The greatest torque is obtained when a force is applied ______ to a wrench.

Q4-5 The vector product of two vectors points ______ to the plane of those vectors.

Q4-6 According to our sign convention, clockwise torques are ______ and counterclockwise torques are ______.

Q4-7 A pair of forces with equal magnitudes but opposite directions is called a ______.

Q4-8 For a rigid body to be in translational equilibrium, the ______ on it must be zero.

Q4-9 For a rigid body to be in rotational equilibrium, the ______ on it must be zero.

Q4-10 The rotational equilibrium condition may be applied about ______.

Q4-11 The weight of an object is effectively concentrated at its ______.

Q4-12 An object is balanced when its C.G. lies above its ______.

Q4-13 The M.A. is the ratio of the ______ to the ______.

Q4-14 When forces are applied perpendicular to a lever, its M.A. is the distance from the fulcrum to the ______ divided by the distance to the ______.

EXERCISES

Section 4.1 | Torques

4-1 Find the magnitude and sign of the torque due to each of the weights in Fig. 4.33 relative to point *P*.

4-2 Find the magnitude and sign of the torque due to each of the weights in Fig. 4.33 relative to point *Q*.

4-3 In Fig. 4.34, consider the vector products **A** × **A**, **A** × **B**, **A** × **C**, **A** × **D**, and **A** × **E**. (a) Which of these vector products are zero? (b) Which are directed into the page? (c) Which are directed out of the page? (d) Which are equal to each other in magnitude and direction?

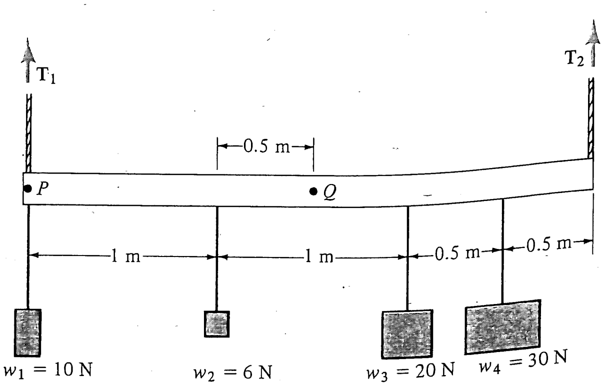

Figure 4.33. Exercises 4-1, 4-2, and 4-11.

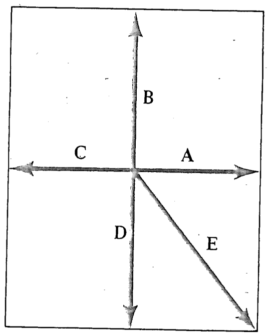

Figure 4.34. The vectors are drawn from the center of a rectangle. Exercises 4-3, 4-4, and 4-5.

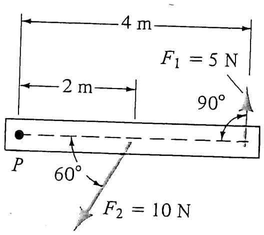

Figure 4.35. Exercises 4-7, 4-8, and 4-10.

4-4 In Fig. 4.34, what are the directions of (a) **B** × **C**; (b) **C** × **B**; (c) **B** × **E**?

4-5 In Fig. 4.34, how is **D** × **A** related to **A** × **E**? Explain.

4-6 Using a wrench 0.4 m long, a force of at least 100 N is needed to turn a nut. (a) How large a torque is required? (b) How large a force is needed to turn the nut using a wrench 0.15 m long?

4-7 In Fig. 4.35, how large are the lever arms for the torques about point P due to $\mathbf{F}_1$ and $\mathbf{F}_2$?

4-8 In Fig. 4.35, find the torques due to $\mathbf{F}_1$ and $\mathbf{F}_2$ relative to point P.

4-9 A cyclist applies a downward force **F** of magnitude 100 N to the pedal of her bicycle (Fig. 4.36). (a) Find the magnitude and direction of the torques in each position shown. (b) In which position is the torque a maximum?

Section 4.2 | Equilibrium of Rigid Bodies

4-10 The bar in Fig. 4.35 is pivoted at point P. Will it tend to start rotating if it is initially at rest? Explain, and indicate in which direction it would rotate if your answer is yes.

4-11 A weightless bar supported by two vertical ropes has four weights hung from it (Fig. 4.33). Find the tensions $\mathbf{T}_1$ and $\mathbf{T}_2$ in the ropes.

4-12 Two children balance on a weightless seesaw. One weighs 160 N and is seated 1.5 m from the fulcrum. The second is seated 2 m on the other side of the fulcrum. What is the weight of the second child?

4-13 Find the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ on the tooth in Fig. 4.37. (In orthodonture, forces applied to the teeth lead to forces on the supporting bones. Gradually the bone tissue breaks down and permits the tooth to rotate or translate. New bone tissue grows in the space left behind. The forces must be small enough to avoid damaging the root of the tooth.)

4-14 Figure 4.38 shows the forearm considered in Example 4.6 when the person is holding a 12-N weight $\mathbf{w}_1$ in the hand (**w** is the weight of the fore-

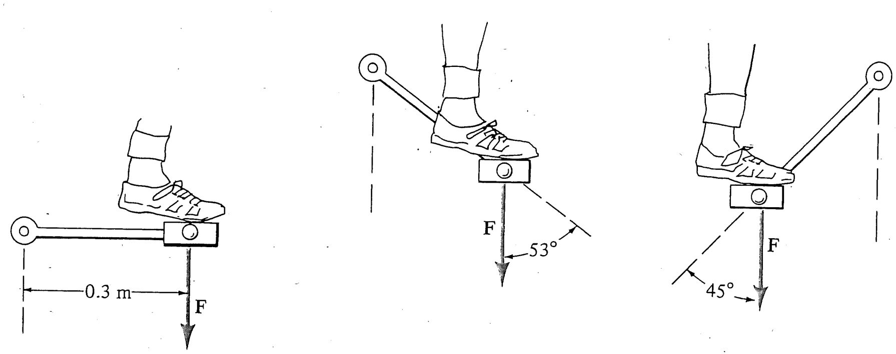

Figure 4.36. Exercise 4-9. (a) (b) (c)

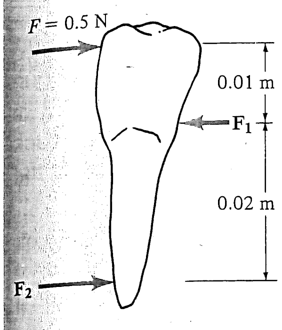

Figure 4.37. Exercise 4-13.

arm). (a) Find the force **T** exerted by the biceps muscle and the force **E** exerted by the elbow joint. (b) In Example 4.6, with $w_1 = 0$, we found $T = 36$ N and $E = 24$ N. Why are these forces more than twice as large here?

4-15 Children with weights w_1 and w_2 balance on a seesaw. The weight w of the seesaw can be considered to act at its center of gravity, which is directly over the pivot. In terms of w, w_1, and w_2, find (a) the force exerted by the pivot; and (b) the ratio x_2/x_1 of the children's distances from the pivot.

4-16 A heavy load of wet laundry is hung from a clothesline. Is the line more likely to break if it is stretched tightly or allowed to sag considerably? Explain.

Section 4.3 | The Center of Gravity

4-17 Three weights are positioned on a weightless rod, as shown in Fig. 4.39. Where is their center of gravity?

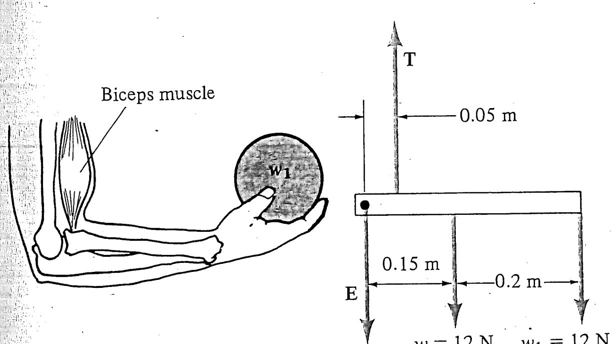

Figure 4.38. Exercises 4-14 and 4-36.

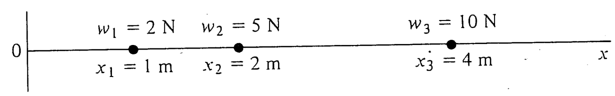

Figure 4.39. Exercise 4-17.

4-18 Two weights are hung from the ends of a horizontal metre stick. If the weight at $x = 0$ is 10 N and the center of gravity is at $x = 0.8$ m, what is the weight at $x = 1$ m? (Neglect the weight of the stick.)

4-19 A woman's forearm has a mass of 1.1 kg and her upper arm has a mass of 1.3 kg. When her arm is held straight out, the C.G. of the forearm is 0.3 m from the shoulder joint, and the C.G. of the upper arm is 0.07 m from the shoulder joint. What is the position of the center of gravity of the entire arm with respect to the shoulder joint?

4-20 An 80-kg hiker carries a 20-kg pack. The center of gravity of the hiker is 1.1 m above the ground when he is not wearing the pack. The C.G. of the pack is 1.3 m from the ground when it is worn. How far above the ground is the C.G. of the hiker and the pack?

4-21 The axles of a car are 3 m apart. The front wheels support a total weight of 9000 N and the rear wheels support 7000 N. How far is the center of gravity from the front axle?

4-22 Using the data on the inside rear cover, find the position of the center of mass of the earth–moon system.

4-23 Two metre sticks are glued together at their ends. One extends from $x = 0$ to $x = 1$ m and the other from $y = 0$ to $y = 1$ m. (a) Where is their center of gravity? (b) How far is the C.G. from the origin?

4-24 A sheet of plywood is 2 m by 3 m. (a) Where is its center of gravity? (b) A 1-m square is removed from one of its corners. Where is the C.G. now, relative to the corner opposite the one where the square was removed?

4-25 A table consists of a thin top plus four legs of constant thickness and of length 0.7 m. If the C.G. of the table is 0.05 m below the top, find the ratio of the mass of the top to the total mass of the legs.

4-26 How large a mass m can be hung from the end of the boom in Fig. 4.40 before it will tip

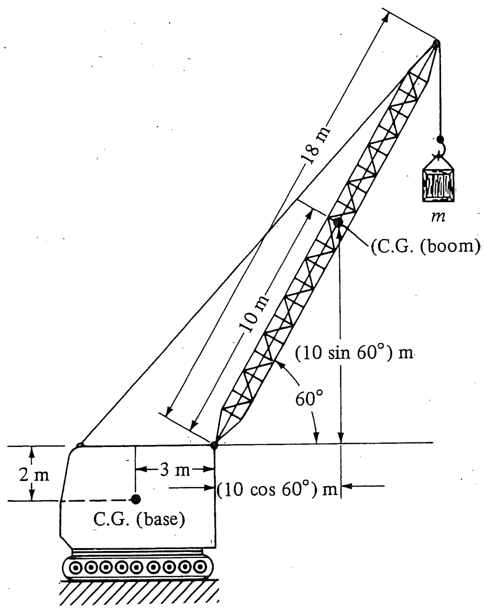

Figure 4.40. Exercise 4-26.

over? (The mass of the movable boom is 1000 kg, and the mass of the base is 10,000 kg.)

4-27 At what angle θ will the table in Fig. 4.41 tip over?

4-28 Ships returning to their home ports without a cargo are sometimes loaded with rocks in their holds or water in their tanks. Why is this done?

4-29 An amusing toy consists of a figure holding a curved pole with weights at either end (Fig. 4.42). It is stable in the position shown and will not fall when pushed gently. Explain why. (*Hint*: Where is the center of gravity?)

4-30 A steel girder with a mass of 1000 kg and a length of 10 m rests on a concrete slab, with 4 m overhanging the edge. How far can a 100-kg man walk on the girder?

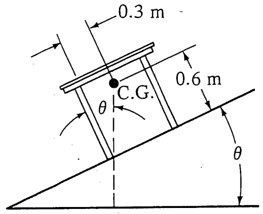

Figure 4.41. Exercise 4-27.

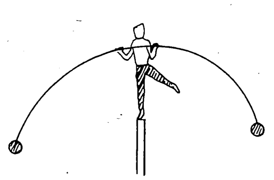

Figure 4.42. Exercise 4-29.

Section 4.5 | Levers; Mechanical Advantage

4-31 A man places a 2-m-long bar under a boulder weighing 4500 N. He uses a fulcrum 0.2 m from the point where the bar touches the rock (Fig. 4.43). What force **F** must he exert to lift the rock?

4-32 An oar is held 0.4 m from the oarlock (Fig. 4.44). If it contacts the water at an average of 1.4 m from the oarlock, what is its mechanical advantage?

4-33 Figure 4.45 shows a pair of tweezers. What is its mechanical advantage?

4-34 Figure 4.46 shows a pair of pliers. (a) What is its mechanical advantage? (b) If a force F =

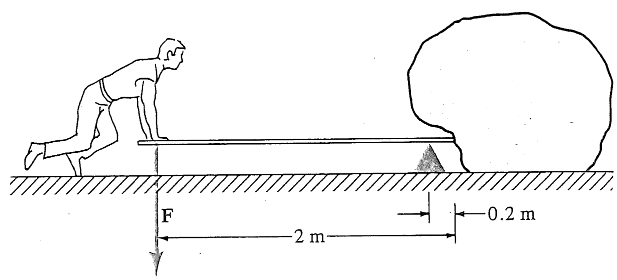

Figure 4.43. Exercise 4-31.

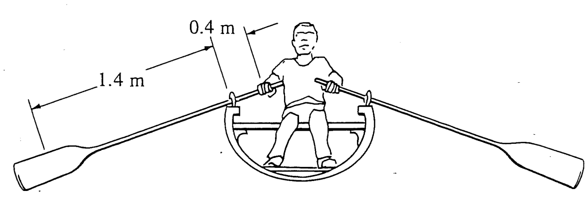

Figure 4.44. Exercise 4-32.

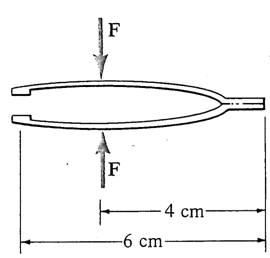

Figure 4.45. Exercise 4-33.

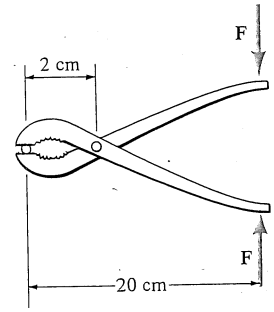

Figure 4.46. Exercise 4-34.

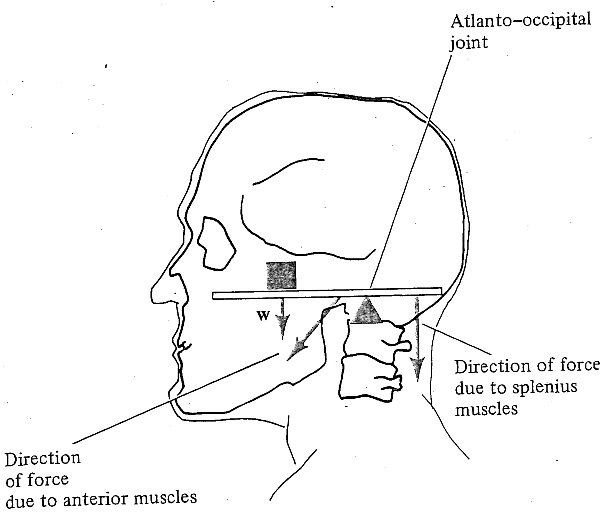

Figure 4.47. Muscles moving and supporting the head. Exercise 4-37.

10 N is applied, what force is exerted on the object?

4-35 Give examples of class I levers with M.A.s equal to 1, less than 1, and greater than 1.

Section 4.7 | Levers in the Body

4-36 Figure 4.38 shows the forearm represented as a pivoted bar; T is the force exerted by the biceps muscle. (a) What class of lever does this represent? (b) What is the mechanical advantage of the forearm for supporting its own weight, w? (c) What is its M.A. for supporting a load w_1 held in the hand? (d) If the muscle contracts 1 cm, how far will the load in the hand move?

4-37 The head pivots about the atlanto-occipital joint (Fig. 4.47). The splenius muscles attached behind the joint support the head. (a) What class of lever does this represent? (b) The anterior muscles produce forward motions of the head. What class of levers does their action represent? (c) Which muscles have the larger mechanical advantage? Speculate on the reasons for this.

PROBLEMS

4-38 Vector **A** points north, and vector **C** = **A** × **B** points straight up. What can you say about (a) the vertical component of **B**; (b) the component of **B** toward the east; (c) the component of **B** toward the north?

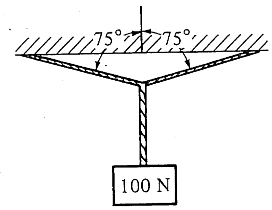

Figure 4.48. Problem 4-39.

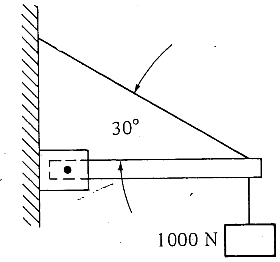

Figure 4.49. Problems 4-40 and 4-41.

4-39 Find the tension in the ropes in Fig. 4.48.

4-40 In Fig. 4.49, an object is supported by a hinged, weightless rod and a cable. Find the tension in the cable and the force exerted by the hinge.

4-41 Repeat the preceding problem if the weight of the rod is 1000 N.

4-42 In Fig. 4.50, the hinged rod and cable are weightless. The cable will break when the tension exceeds 2000 N. What is the maximum weight w that can be supported?

***4-43** The tension T at each end of the chain in Fig. 4.51 is 20 N. What is the weight of the chain?

4-44 A horse stands with its left forefoot off the ground (Fig. 4.52). The left rear and right front legs each support 1500 N of its weight, which is 5000 N. (a) What force is exerted by the right hind

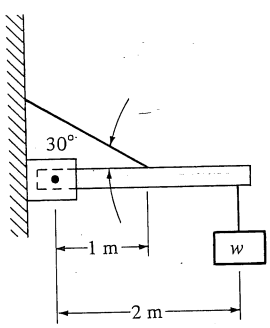

Figure 4.50. Problem 4-42.

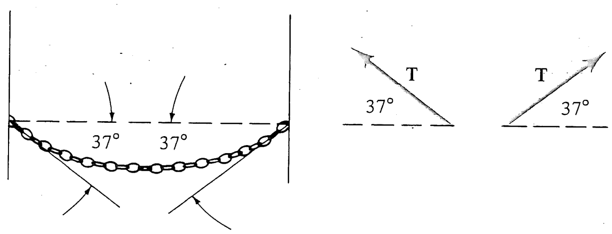

Figure 4.51. Problem 4-43.

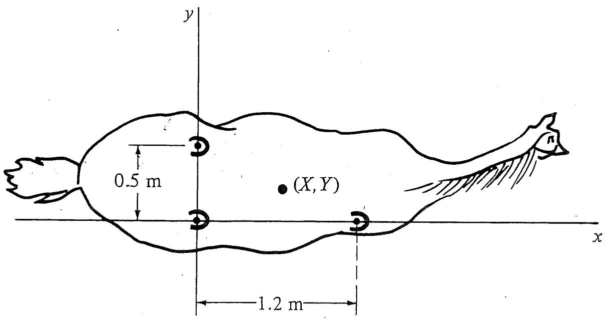

Figure 4.52. A horse viewed from above standing on three legs. Problem 4-44.

leg? (b) Find the position (X, Y) of the center of gravity.

4-45 What is the position of the center of mass of the three masses of Fig. 4.53?

4-46 The man in Fig. 4.54 has a mass of 100 kg. His arms are held straight out to the side, and in one hand he holds a mass M. (a) Find the horizontal and vertical positions of the C.G. of the man plus the mass M. (Choose the origin to be midway between his feet). (b) What is the maximum mass the man can hold without falling over?

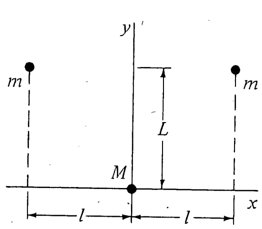

Figure 4.53. Problem 4-45.

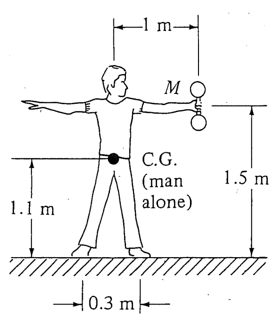

Figure 4.54. Problem 4-46.

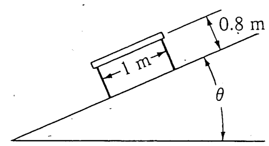

Figure 4.55. Problems 4-47 and 4-48.

***4-47** A four-legged table has a top of mass 20 kg. The legs are positioned at the corners of the top and have a mass of 2 kg each. The dimensions of the table are shown in Fig. 4.55. At what angle θ will the table tip over?

4-48 The table in Fig. 4.55 has massless legs. At what angle θ will it tip over?

4-49 A uniform wooden board has a mass of 20 kg and a length of 2 m. A circular hole is cut out with its center 0.5 m from one end. If the C.G. is now 0.9 m from the opposite end, what is the mass of the wood removed?

***4-50** The ammonia (NH_3) molecule is a pyramid, with the three hydrogen (H) atoms forming the base and the nitrogen (N) atom at the apex. The centers of the hydrogen atoms are separated by 16.3 nm (1 nm $= 10^{-9}$ m), and the nitrogen atom is 3.8 nm above the center of the base. Where is the center of mass of the molecule relative to the nitrogen atom? (The mass of a nitrogen atom is 14 times that of a hydrogen atom.)

***4-51** The deltoid muscle raises the upper arm to a horizontal position (Fig. 4.56). (a) Find the tension T exerted by the muscle and the components R_x and R_y of the force exerted by the shoulder joint. (b) What is the mechanical advantage of the muscle for lifting the arm?

***4-52** A vase 0.4 m tall has its center of gravity 0.15 m from the bottom, which is a circle of radius 0.05 m (Fig. 4.57). How far can the top of the vase be pushed to the side without toppling it?

4-53 Show that the M.A. of a class III lever is always less than 1, assuming the forces are perpendicular to the lever.

4-54 Show that the M.A. of a class II lever is

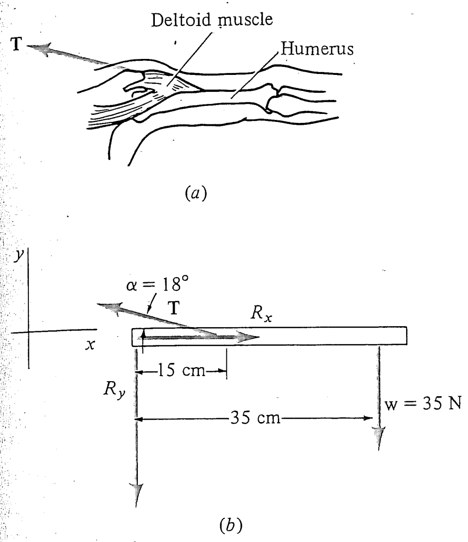

Figure 4.56. Problem 4-51. (Adapted from Williams and Lissner.)

always greater than 1, assuming the forces are perpendicular to the lever.

***4-55** In Fig. 4.58, the weight of the upper body is w = 490 N. Find the force T exerted by the spinal muscles and the components R_x and R_y of the force R exerted by the pivot (sacrum) if the weight w_1 is (a) zero; (b) 175 N.

4-56 Show that 1 N m = 1 kg m² s⁻².

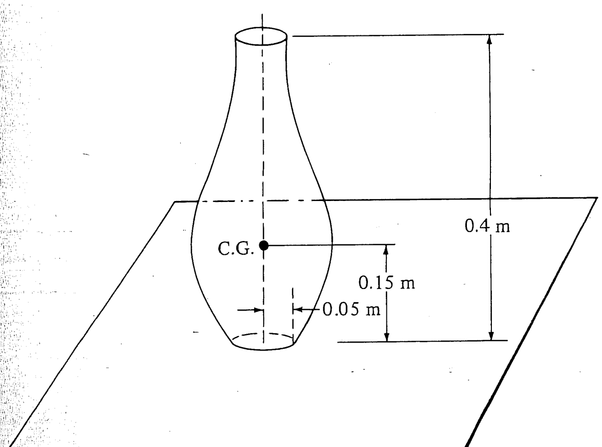

Figure 4.57. Problem 4-52.

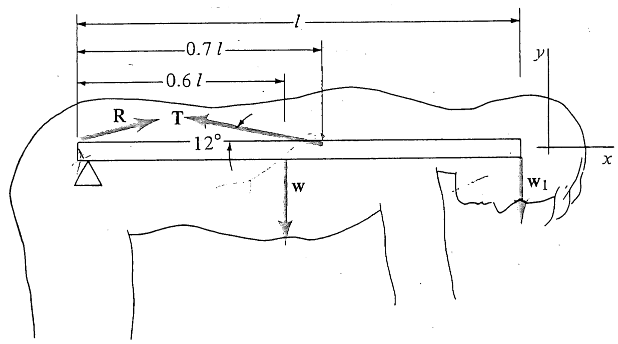

Figure 4.58. Problem 4-55.

ᶜ4-57 A rod is shaped so that its weight per unit length λ is $\lambda_0(1 - x/2L)$ for $0 \leq x \leq L$. Find its center of gravity.

ᶜ4-58 Find the center of gravity of the triangular metal plate in Fig. 4.59.

ᶜ4-59 A solid cone of uniform density has a height h (Fig. 4.60). (a) Show that its weight per

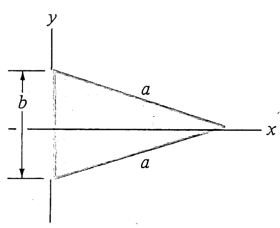

Figure 4.59. Problem 4-58.

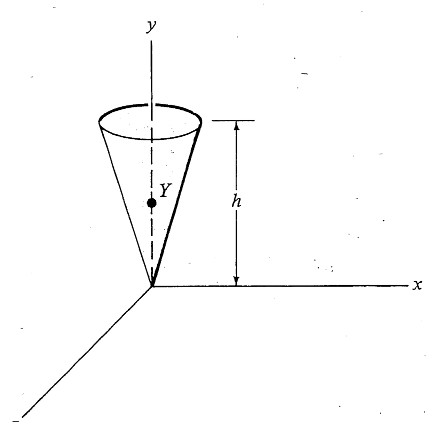

Figure 4.60. Problem 4-59.

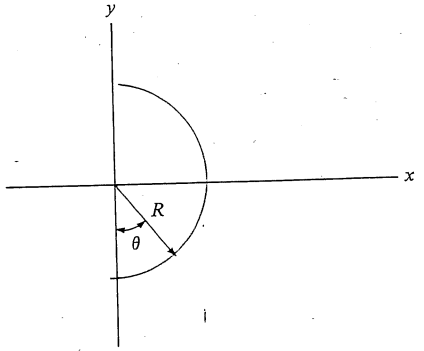

Figure 4.61. Problem 4-60.

unit length alone the y direction varies as y^2. (b) Find its center of gravity, Y.

c4-60 A wire is bent into a semicircle of radius R (Fig. 4.61). Find its center of gravity. [*Hint:* Write $\int x\, dw$ in terms of the angle θ.

ANSWERS TO REVIEW QUESTIONS

Q4-1, size or shape; **Q4-2**, torque; **Q4-3**, lever arm; **Q4-4**, perpendicular; **Q4-5**, perpendicular; **Q4-6**, negative, positive; **Q4-7**, couple; **Q4-8**, net force; **Q4-9**, net torque; **Q4-10**, any convenient point; **Q4-11**, center of gravity; **Q4-12**, base; **Q4-13**, load force, applied force; **Q4-14**, point where applied force acts, load.

SUPPLEMENTARY TOPICS

4.8 | THE JAWS OF ANIMALS

Mechanics enables us to understand why many anatomical structures have evolved to their present state, since the mechanical functions of bones, muscles, and joints largely determine their sizes and shapes. This is illustrated nicely by the development of the lower jaws of mammals.

It is often advantageous for an animal to be able to bite very hard. The biting force depends on the magnitude, direction, and point of application of the forces exerted by the muscles closing the jaw. This leads to certain optimal shapes and sizes of jaws. In addition, the bones of the jaw joint connecting the upper and lower jaws must be strong enough to prevent fractures and dislocations. From fossil records, we know that mammals evolved from mammal-like reptiles. As this occurred, the muscles attached to the lower jaw progressively *increased* in size, while the bones forming the jaw joint steadily *decreased* in size. This apparent paradox can be explained in terms of the changes in the direction and point of application of the muscular forces.

Figure 4.62 shows the basic differences between the lower jaws of a primitive reptile and a typical present-day mammal. The first is a simple bar, with the muscles pulling upward at a point close to the joint. The mammalian jaw has a large bump or projection called the *coronoid process*. Attached to this is the *temporalis* muscle, which pulls *backward* as well as upward (force **T** in Fig. 4.62). The *masseter* and *pterygoideus* muscles pull *forward* as well as upward (force **M**).

A primitive reptile biting with an upward force −**B** on food between its back teeth experiences an equal but opposite reaction force **B** downward on the jaw. Since the muscular force **M** is applied close to the joint, static equilibrium can be achieved only if a large downward force **R** is exerted by the joint.

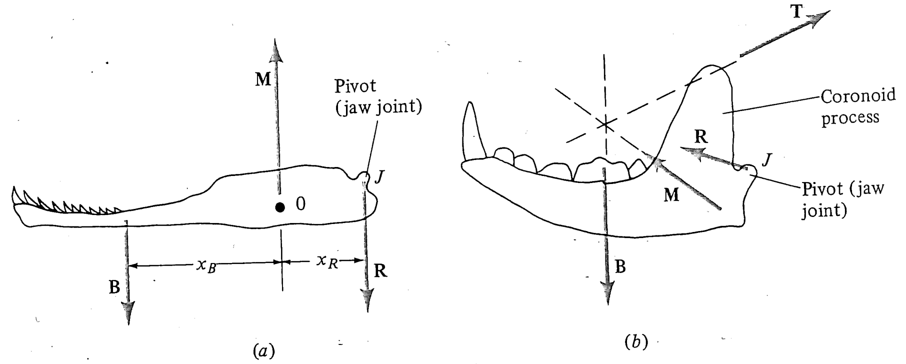

Figure 4.62. (*a*) Lower jaw of a primitive reptile. M is the force due to the muscle, B is the reaction force from the object being bitten, and R is the force due to the jaw joint at J. (*b*) A mammalian jaw. Muscle forces are shown as T and M. As explained in the text, the force R due to the jaw joint can be zero if the lines of action of the three forces T, B, and M intersect as shown here.

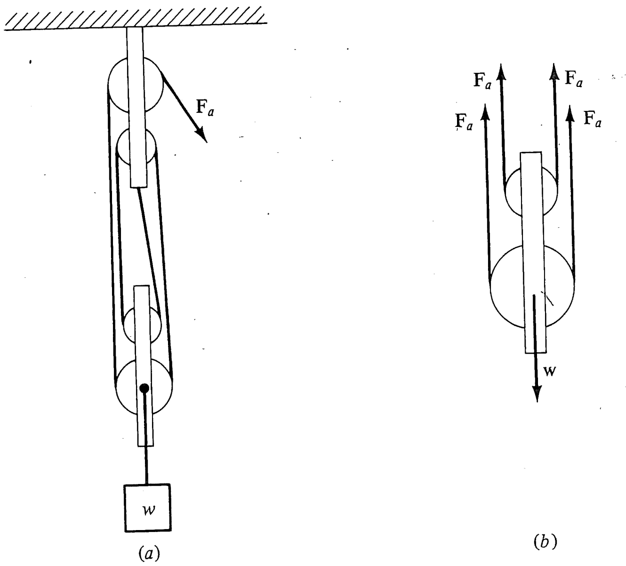

Figure 4.68. Example 4.14.

From the preceding two examples, we can infer a rule for the mechanical advantage of pulley systems used to lift weights. *The mechanical advantage of the system is equal to the number of parallel ropes supporting the pulley to which the load is attached*: 2 in Fig. 4.67 and 4 in Fig. 4.68. Note that this rule does *not* apply when, as in the next example, the forces applied to the load are not all parallel.

Example 4.15

Leg traction is applied to a patient's leg as shown in Fig. 4.69. What horizontal force is exerted on the leg?

The sum of the forces on each pulley is zero, since the pulleys are at rest. From Fig. 4.69*b*, the horizontal forces that act on the pulley attached to the foot satisfy

$$2w \cos \theta - F_L = 0$$

or

$$F_L = 2w \cos \theta$$

This force can be changed by altering either w or θ. Since $\cos \theta$ varies from 1 to 0 as θ goes from 0° to 90°, any force from zero to $2w$ can be obtained by choosing the correct angle. When θ is large, $\cos \theta$ is small. Then the weight w and the tension in the rope are much larger than the force F_L exerted on the foot.

EXERCISES ON SUPPLEMENTARY TOPICS

Section 4.8 | The Jaws of Animals

4-61 A snake exerts a muscle force $M = 5$ N (see Fig. 4.62*a*). M acts at a distance of 0.03 m from the joint, and the resulting bite force is 2 N. Find (a) the distance from the joint to the line of action of the bite force; (b) the force exerted by the jaw joint.

4-62 (a) In a typical herbivore, the maximum magnitude of the force T is one tenth the maximum magnitude of the force M in Fig. 4.70. Assuming that there is no force at the joint, would you expect the animal to exert the largest biting force near the front or back of the jaw? (b) In a carnivore the maximum value of T is about twice that of M. Would you expect the maximum biting force to be exerted further from or closer to the jaw joint than in the herbivore? Explain.

Section 4.9 | Center of Gravity of Humans

4-63 A board 4 m long is used to find the C.G. as in Fig. 4.64. When a person is on the board, the

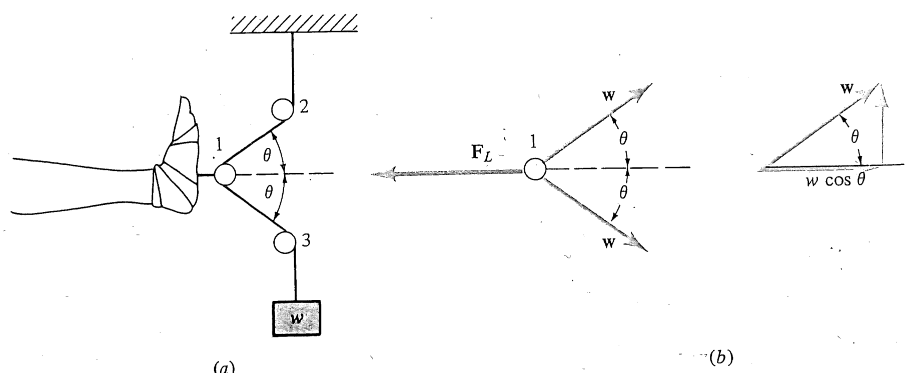

Figure 4.69. (*a*) A system of pulleys used to apply a force in leg traction. The magnitude of this force can be adjusted by changing the angle θ. Pulley 1 is attached to the foot, and pulleys 2 and 3 are mounted on a rigid frame that is not shown. (*b*) The forces on pulley 1.

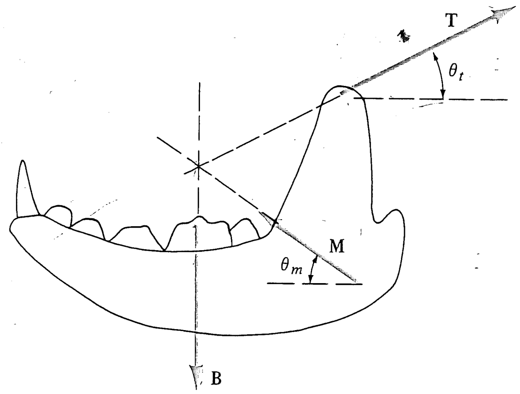

Figure 4.70. Exercise 4-62; Problems 4-69 and 4-70.

scale readings are $w_1 = 200$ N and $w_2 = 600$ N. What is the position of the C.G. of the person?

4-64 Using the data of Table 4.1, find the C.G. of the man in Fig. 4.65.

4-65 Using the data of Table 4.1, find the C.G. of the man in Fig. 4.66.

Section 4.10 | Pulley Systems

4-66 What force F must be applied in Fig. 4.71 to lift the load?

4-67 In Fig. 4.69, a 50-N force is to be applied to the leg. If a 10-kg mass is hung from the cable, what angle θ should be used?

4-68 Suppose the rope in Fig. 4.68 is pulled at 0.25 m s^{-1}. How fast will the load rise?

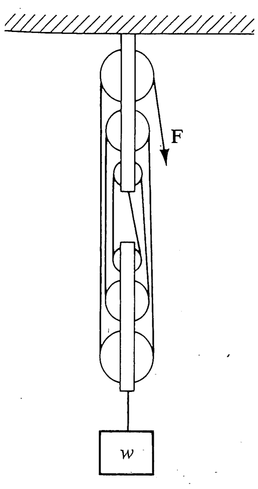

Figure 4.71. Exercise 4-66.

PROBLEMS ON SUPPLEMENTARY TOPICS

4-69 A mammal bites so that the muscle force **M** of Fig. 4.70 has a magnitude of 30 N. What is the force **B** of the bite? (Assume $\theta_t = \theta_m = 45°$.)

***4-70** In a particular carnivore the magnitude of the force T is 1.3 times the magnitude of M (Fig. 4.70). There is no force at the jaw joint. If $\theta_m = 60°$, find (a) θ_t; (b) the ratio B/M.

Additional Reading

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