vector components
unit vector
velocity in two dimensions
acceleration in two dimensions
projectile

REVIEW QUESTIONS

Q2-1 What is the difference between a scalar and a vector?

Q2-2 How do we represent vectors in writing and in textbooks?

Q2-3 If **A** is parallel to **B**, what is the magnitude of **A** + **B**? Of **A** − **B**? Of 2**A**?

Q2-4 If **A** is perpendicular to **B**, what is the magnitude of **A** + **B**?

Q2-5 The average velocity in a plane is the ______ divided by the ______.

Q2-6 The average acceleration in a plane is the ______ divided by the ______.

Q2-7 The acceleration is zero when both the ______ and ______ of the velocity are constant.

Q2-8 Motion in a plane is equivalent to a pair of ______.

Q2-9 The motion of a projectile is influenced only by the ______ (assuming air resistance is negligible).

Q2-10 The time when a projectile hits the ground is found from the equations for the ______ motion.

Q2-11 When a projectile is at its greatest height, the ______ component of the velocity is zero.

Q2-12 The ______ component of the velocity of a projectile remains constant throughout its motion.

Q2-13 The vertical component of the acceleration of a projectile is ______, and the horizontal component is ______.

EXERCISES

Section 2.1 | An Introduction to Vectors

2-1 Figure 2.19 shows a collection of vectors that can be combined in various ways. For example, **A** + **C** = **B**. Find (a) **E** + **C**; (b) **A** + **F**; (c) **A** + **D**; (d) **E** + **A**; (e) **E** + 2**A**; (f) **A** − **B**; (g) **B** − **A**; (h) **C** − **A**.

2-2 In Fig. 2.20, for what value of θ will **C** = **A** + **B** have (a) a minimum magnitude, and (b) a maximum magnitude? (c) Find C when $\theta = 90°$.

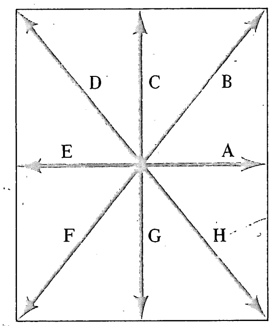

Figure 2.19. Exercise 2-1.

2-3 A vector has an x component of -10 and a y component of $+3$. (a) Draw a set of x–y axes and show the vector. (b) Calculate the magnitude and direction of the vector.

2-4 $\mathbf{A} = 3\hat{\mathbf{x}} + 2\hat{\mathbf{y}}$, and $\mathbf{B} = 4\hat{\mathbf{x}} - \hat{\mathbf{y}}$. Find the magnitude of (a) **A** + **B**; (b) **A** − **B**; (c) 2**B**.

2-5 $\mathbf{A} = 2\hat{\mathbf{x}} + 4\hat{\mathbf{y}}$. Find the magnitude and direction of (a) **A**; (b) −**A**.

2-6 If $\theta = 72°$ in Fig. 2.20, find (a) the direction and magnitude of **C** = **A** + **B** by constructing a drawing using a ruler and a protractor; (b) the direction and magnitude of **C** using the component method.

2-7 For the vectors **A** and **B** in Fig. 2.21, find (a) **A** + **B**; (b) **B** − **A**; (c) **A** − **B**.

2-8 Using components for the vectors in Fig. 2.22, find the direction and magnitude of **E** = **A** + **B** + **C** + **D**.

2-9 Using components for the vectors in Fig. 2.22, find the direction and magnitude of **F** = **A** − **C** + **B** − 2**D**.

2-10 A woman walks 10 km north, turns toward the northwest, and walks 5 km further. What is her final position?

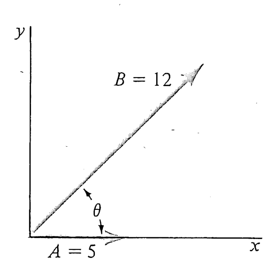

Figure 2.20. Exercises 2-2 and 2-6.

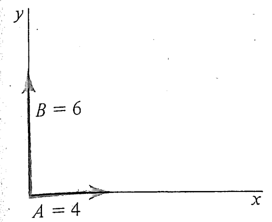

Figure 2.21. Exercise 2-7.

2-11 A ship sets out to sail 100 km north but is blown by a severe storm to a point 200 km east of its starting point. How far must it sail, and in what direction, to reach its intended destination?
2-12 One person walks northeast at 3 km h^{-1}, and another heads south at 4 km h^{-1}. How far apart are they after 2 h?
2-13 For the vectors in Fig. 2.23, find the magnitude and direction of (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$; (b) $\mathbf{E} = \mathbf{A} - \mathbf{B} - \mathbf{C}$.
2-14 For the vectors in Fig. 2.24, find the magnitude and direction of (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$; (b) $\mathbf{E} = \mathbf{A} - \mathbf{B} - \mathbf{C}$.

Section 2.2 | The Velocity in Two Dimensions

2-15 A car goes around a circular track 500 m in diameter at a constant speed of 20 m s^{-1}. (a) How long does it take for the car to go halfway around the track? (b) What is its average velocity in that time interval?
2-16 A car goes around a circular track with a radius of 1000 m at a constant speed of 10 m s^{-1}. (a) How long does it take the car to go once completely around the track? (b) What is the average velocity of the car over this time interval?

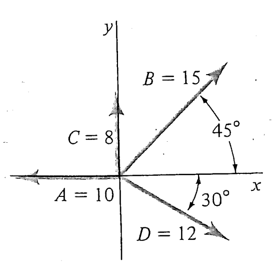

Figure 2.22. Exercises 2-8 and 2.9.

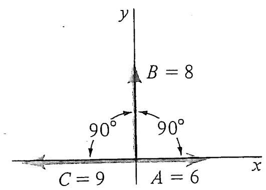

Figure 2.23. Exercise 2-13.

2-17 A ball is thrown at 30 m s^{-1} at an angle of 20° to the horizontal direction. Find the horizontal and vertical components of its initial velocity.
2-18 A plane flies for 3 h and reaches a point 600 km north and 800 km east of its starting point. Find the direction and magnitude of its average velocity.
2-19 A plane flies south at 500 km h^{-1} for 2 h and then flies west at 500 km h^{-1} west for 1 h. (a) What is its average speed? (b) What are the magnitude and direction of its average velocity?
2-20 A ship is sailing north at 10 m s^{-1}. A man runs east across it at 8 m s^{-1}. (a) How far has he moved relative to the water after 3 s? (b) In what direction has he moved?
[c]**2-21** An object has its position given by the formula $r = pt\mathbf{x} + qt^2\mathbf{y}$, where p and q are constants. Find its instantaneous velocity components as functions of time.

Section 2.3 | The Acceleration in Two Dimensions

2-22 A car initially traveling due north goes around a semicircle having a radius of 500 m at a constant speed of 20 m s^{-1}. (a) How long does this take? (b) What is the magnitude and direction of the average acceleration?

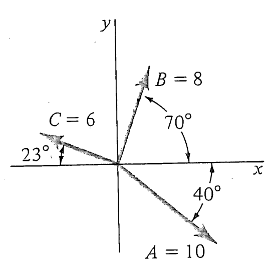

Figure 2.24. Exercise 2-14.

2-23 A rifle pointed at 30° to the horizontal fires a bullet at 250 m s^{-1}. If the bullet is accelerated uniformly in the barrel for 0.006 s, find (a) the magnitude of the acceleration; (b) its horizontal and vertical components.

2-24 A tennis ball is served by a player, bounces in the opponent's court, and is hit by the opponent toward the first player. Describe the direction and magnitude of the acceleration during each part of the motion.

2-25 The earth rotates about the sun once each year on an approximately circular path. Find the magnitude of the average acceleration associated with this motion over a 6-month interval. (The average distance from the earth to the sun is 1.50×10^{11} m.)

c**2-26** Find the instantaneous acceleration of the object in Exercise 2-21.

Section 2.4 | Finding the Motion of an Object and Section 2.5 | Projectiles

2-27 A ball is thrown from a rooftop at a 45° angle above the horizontal and a few seconds later hits the ground. At what point in its motion does it have its (a) greatest velocity; (b) smallest velocity; (c) greatest acceleration?

2-28 A soccer ball is kicked into the air. What is its acceleration (a) as it rises; (b) at the top; (c) on the way down?

2-29 A hunter aims a rifle at an angle of 20° above the horizontal. She fires a bullet while simultaneously dropping another bullet from the level of the rifle. Which bullet will hit the ground first?

2-30 An inspired physics teacher runs across a lecture table and right off the end. Exactly 0.5 s later, she lands on the floor 2 m from the end of the table. Neglect air resistance, and assume her velocity was horizontal as she left the table. (a) What was her velocity as she left the table? (b) What is the height of the table?

2-31 A baseball player measures his pitching velocity by throwing horizontally from a height of 5 m above the ground. The ball hits the ground 25 m away. What is his pitching velocity?

2-32 A cannon fires a projectile at 100 m s^{-1} at an angle of 30° above the horizontal. How high will the projectile be when its horizontal distance from the cannon is 200 m?

2-33 A football kicked into the air from ground level hits the ground 30 m from where it started after 4 s. (a) Find its average velocity while in the air. (b) Find its average acceleration while in the air.

2-34 A ball is thrown horizontally at 20 m s^{-1} from a window 15 m above the ground. (a) When will it hit the ground? (b) Where will it land?

2-35 A rifle pointed at 30° above the horizontal fires a bullet at 500 m s^{-1}. The rifle barrel is 0.7 m long. (a) Find the average acceleration in the rifle barrel. (b) Find the horizontal and vertical components of this acceleration.

2-36 A snowball is thrown from 2 m above the ground at a velocity of 10 m s^{-1} directed at 30° above the horizontal. (a) Find its horizontal and vertical position after 1 s. (b) Find its velocity components after 1 s.

2-37 (a) How long will the snowball of the preceding exercise be in the air? (b) Where will it land?

2-38 A baseball is hit at 40 m s^{-1} at an angle of 30° to the horizontal. (a) How high will it go? (b) When will it reach that height? (c) What will be its horizontal distance from the batter at that time?

2-39 For the baseball in the preceding exercise, find (a) the horizontal distance it will travel; (b) the total time it will be in the air. (Neglect the fact that the ball is struck from slightly above ground.)

2-40 A rifle is aimed directly at a target 200 m away at the same height as the rifle. If the bullet leaves the muzzle at 500 m s^{-1}, by how much will it miss the target?

2-41 The earth revolves about its axis every 24 hours. Find the magnitude of the average acceleration of a point on the equator over a 6-hour time interval. (The radius of the earth is 6.38×10^{6} m.)

PROBLEMS

2-42 Suppose that in Fig. 2.15 the stuffed animal is initially 1 m above and 1.5 m to the right of the cannon that is pointed at the animal. The animal starts to fall as the cannonball is fired at 10 m s^{-1}.

(a) What are the initial horizontal and vertical velocity components of the cannonball? (b) How long does it take for the horizontal coordinate of the ball to change by 1.5 m? (c) What are the vertical positions of the ball and the animal at that time?

2-43 One ball is thrown horizontally with a velocity v_0 from a height h, and another is thrown straight down with the same initial speed. (a) Which ball will land first? (b) Which ball will have the greater speed as it is about to land?

2-44 A boy throws a ball so that it rises 1 m while traveling 7 m horizontally and then begins to drop. What were the initial speed and direction of the ball?

2-45 A man points his boat due east and rows at 4 km h^{-1} relative to the water. The river flows south at 2 km h^{-1}, and it is 1 km wide. (a) How fast is the man moving relative to the shore? (b) Where will he land? (c) How long will his trip take?

2-46 A woman wishes to go directly across a river that is 1 km wide. The river flows south at 2 km h^{-1}, and she can row at 4 km h^{-1} relative to the water. (a) In what direction should she row relative to the water? (b) How long will it take her to reach the other side?

2-47 A snowball is thrown from the ground at a building 30 m away. It hits the building 20 m above the height from which it was thrown. Its initial horizontal velocity component was 30 m s^{-1}. What was the initial vertical velocity component?

2-48 A tennis ball is served horizontally at a height of 2.4 m, 12 m from a net that is 0.9 m high. (a) If it is to clear the net by at least 0.2 m, what is its minimum initial velocity? (Neglect air resistance.) (b) If it clears the net by 0.2 m, where will it land?

2-49 Some books advise serving a tennis ball at an angle below the horizontal direction. To see if this is sound advice, suppose a ball is struck at an angle of 5° downward at a height of 2.4 m with the relatively high speed of 30 m s^{-1}. How high will it be when it reaches the net 12 m away? (The net is 0.9 m high. Neglect air resistance.)

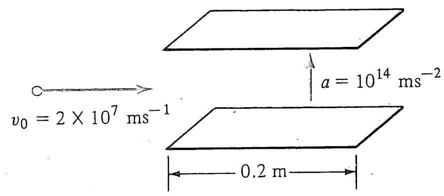

Figure 2.25. Problem 2-50.

2-50 The screens of cathode ray tubes in television sets and oscilloscopes emit light when they are struck by rapidly moving electrons. Electrical deflecting plates are used to control where the electrons strike. In Fig. 2.25, electrons with an initial horizontal velocity of 2×10^7 m s^{-1} experience a vertical acceleration of 10^{14} m s^{-2} while they are between the plates, which are 0.2 m long. (a) How long will the electrons be between the plates? (b) In what direction will the electrons be moving after they leave the plates? (c) How far will the electrons be deflected vertically as they leave the plates?

***2-51** A tennis ball is served 2.5 m above the ground at an angle of 5° above the horizontal direction with an initial speed of 30 m s^{-1}. (a) When will it hit the ground? (b) How far will it travel?

2-52 A ski jumper leaves a slope at an angle of 20° above the horizontal direction. She lands 3.5 s later at a point 20 m below her takeoff point. (a) What was her initial speed? (b) How far does she travel horizontally?

2-53 A boy standing 10 m from a building can just barely reach the roof 12 m above him when he throws a ball at the optimum angle with respect to the ground. Find the initial velocity components of the ball.

2-54 Derive a formula for the maximum height reached by a projectile in terms of its initial velocity components.

***2-55** Show that the horizontal and vertical displacement components for a projectile satisfy an equation of the form $\Delta y = a\,\Delta x + b(\Delta x)^2$, so that the trajectory is a *parabola*.

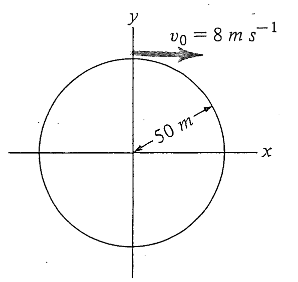

Figure 2.26. Problem 2-57.

[c]**2-56** A point on the rim of a rolling wheel has its position given by $x = R \sin \omega t + \omega R t$, $y = R \cos \omega t + R$. (a) Sketch the path followed by the point. (This curve is called a cycloid.) (b) Find the instantaneous velocity and acceleration components at time t. (c) Find the magnitudes of v and a at time t.

[c]**2-57** A runner goes around a circular track of radius 50 m at a constant speed of 8 m s^{-1} (Fig. 2.26). Assume she starts to run clockwise at $t = 0$ s from $x = 0$ m, $y = 50$ m. For $t > 0$, find the components of her (a) position; (b) velocity; (c) acceleration.

[c]**2-58** (a) Show that the acceleration of an object undergoing circular motion at constant speed is directed toward the center of the circle. (b) Is this true if the speed is not constant? Explain.

[c]**2-59** An object has an acceleration $\mathbf{a} = p\hat{\mathbf{x}} + qt\hat{\mathbf{y}}$, where p and q are constants. It starts from rest at the origin at $t = 0$. Using integration techniques (Sec. 1.9), for $t > 0$ find (a) its velocity; (b) its position.

ANSWERS TO REVIEW QUESTIONS

Q2-1, scalar has magnitude only, vector also has direction; **Q2-2**, arrows over symbols, boldface type; **Q2-3**, $A + B$, $A - B$, $2A$; **Q2-4**, $(A^2 + B^2)^{1/2}$; **Q2-5**, displacement, elapsed time; **Q2-6**, velocity change, elapsed time; **Q2-7**, magnitude, direction; **Q2-8**, one-dimensional motions; **Q2-9**, gravitational acceleration; **Q2-10**, vertical; **Q2-11**, vertical; **Q2-12**, horizontal, **Q2-13**, $-g$, 0.

SUPPLEMENTARY TOPICS

2.6 | PROJECTILES IN BIOMECHANICS

Many applications of projectile motion occur in athletics and in animal motion. Here we explore some further aspects of this subject.

In applications of projectile motion, it is convenient to have a formula for the horizontal distance traveled or *range*, R. To obtain this formula, consider a projectile launched from a flat surface (Fig. 2.27). The projectile lands after an elapsed time Δt when Δy returns to zero. The range can be found from the equation for Δx once this elapsed time is known.

With $\Delta y = 0$, we can rewrite $\Delta y = v_{0y}\, \Delta t - \frac{1}{2}g(\Delta t)^2$ as

$$(v_{0y} - \tfrac{1}{2}g\, \Delta t)\Delta t = 0$$

The solutions of this equation are $\Delta t = 0$, which corresponds to the instant the projectile was launched, and

$$\Delta t = \frac{2v_{0y}}{g} \tag{2.17}$$

which gives the elapsed time the projectile is in motion.

Using this elapsed time in $\Delta x = v_{0x}\, \Delta t$, the range is

$$R = \frac{2v_{0x}v_{0y}}{g}$$

If the initial velocity of the projectile is at a *launch angle* θ_0 to the ground (Fig. 2.27), $v_{0x} = v_0 \cos \theta_0$ and $v_{0y} = v_0 \sin \theta_0$. Then the range can be expressed as

$$R = \frac{2v_0^2 \sin \theta_0 \cos \theta_0}{g} \tag{2.18}$$

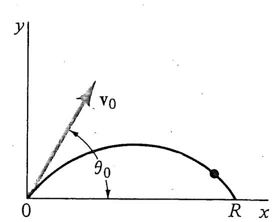

Figure 2.27. A projectile with a velocity $\mathbf{v}_0$ launched at an angle θ_0 to the horizontal has a range R.

The same range can also be obtained with $\theta_0 = 90° - 15° = 75°$, but the elapsed time will be longer.

(c) Using Eq. 2.17 and $v_{0y} = v_0 \sin\theta_0$, the times are

$$\Delta t_a = \frac{2v_{0y}}{g} = \frac{2(36 \text{ m s}^{-1})(\sin 45°)}{9.8 \text{ m s}^{-2}} = 5.20 \text{ s}$$

$$\Delta t_b = \frac{2(36 \text{ m s}^{-1})(\sin 15°)}{9.8 \text{ m s}^{-2}} = 1.90 \text{ s}$$

Note that the elapsed time in part (b) is less than half that in (a), even though the range is halved, because the trajectory is much flatter. The time for *two* of the shorter throws is less than Δt_a by (5.20 s) − 2(1.90 s) = 1.40 s. Rather than throwing directly to the plate, an outfielder frequently throws to a player who relays the ball to home plate. This makes the accuracy less critical and often saves time. (Allowing for the time to make the second throw reduces the savings, while air resistance effects increase them.)

Horizontal Jumping | In Section 1.8, we saw that the constant acceleration formulas can be used to analyze vertical jumping by animals. Similarly, the projectile motion formulas can be used to discuss horizontal jumping, since they accurately describe the motion while the animal is in the air if air resistance is negligible.

Although a 45° launch angle produces the maximum range on flat ground for a given initial speed (Fig. 2.30), an animal may customarily jump at some other angle for reasons related to its needs or structure. For example, locusts often jump into the air and then start flying. In this case the range of the jump is clearly irrelevant, but the time duration may be significant. Whether or not they begin to fly, locusts usually jump at about 55°.

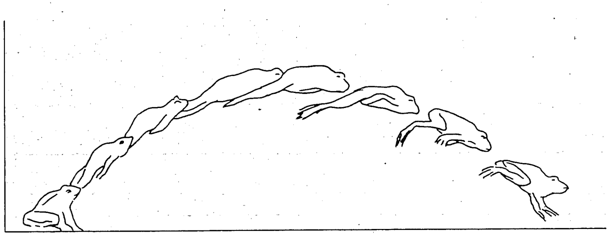

Figure 2.30. Frogs frequently jump with a launch angle of approximately 45°, the angle that produces the maximum range on flat ground.

The calculation of the takeoff velocity is illustrated in the following example.

Example 2.14

What is the takeoff speed of a locust if its launch angle is 55° and its range is 0.8 m?

Since we know R and θ_0, we can find v_0 from

$$R = \frac{2v_0^2}{g}\sin\theta_0\cos\theta_0$$

With sin 55° = 0.819 and cos 55° = 0.574,

$$v_0^2 = \frac{gR}{2\sin\theta_0\cos\theta_0} = \frac{(9.8 \text{ m s}^{-2})(0.8 \text{ m})}{2(0.819)(0.574)}$$
$$= 8.3 \text{ m}^2 \text{ s}^{-2}$$
$$v_0 = 2.9 \text{ m s}^{-1}$$

EXERCISES ON SUPPLEMENTARY TOPICS

Section 2.6 | Projectiles in Biomechanics

2-60 A football is kicked at 20 m s^{-1} from ground level. Find its range if the launch angle is (a) 30°; (b) 60°; (c) 45°.

2-61 In the preceding exercise, how long will the football be in the air in each of the three cases?

2-62 An astronaut wearing his spacesuit can broad jump 2 m on the earth. How far can he jump on a planet where the gravitational acceleration is half that at the surface of the earth?

2-63 A girl wishes to throw a snowball at another child. If she can throw the snowball at 20 m s^{-1}, how far can she stand from the other child and still reach him?

2-64 A rifle is fired horizontally from the top of a tall mountain. Using a sketch, show the effect of the earth's curvature on the range of the bullet.

2-65 A kangaroo can jump 8 m. If it takes off at 45° to the horizontal, what is its takeoff speed?

2-66 A baseball thrown at 10° to the horizontal returns to its original height after 70 m. What was its original speed?

2-67 A motorcycle stunt rider leaves a ramp at 30° to the horizontal, just barely clears a row of trucks 36 m wide, and lands at the same height as his takeoff point. What was his takeoff speed?

2-68 A rifle is aimed slightly above a target 200 m away at the same height as the rifle. The bullet leaves the muzzle at 500 m s^{-1} and strikes the center of the target. At what angle to the horizontal is the rifle barrel?

2-69 A football is kicked 60 m on a level field. If the launch angle is 60°, how large is its initial velocity?

2-70 A rescue ship is to fire a shell trailing a lifeline to a distressed vessel located at a distance of 300 m. The initial velocity of the shell is 100 m s^{-1}. What are the possible launch angles? (Neglect the effect of the trailing lifeline.)

2-71 A mortar shell is fired at a ground-level target 500 m distant with an initial velocity of 90 m s^{-1}. What is its launch angle? (Mortars are fired at large launch angles.)

2-72 A football is kicked from ground level on a level field with an initial velocity v_0 and a launch angle θ_0. Find the velocity v and the angle θ at which it hits the ground.

PROBLEMS ON SUPPLEMENTARY TOPICS

2-73 With the aid of a sketch, show that the maximum range for an object launched at a given speed from below its landing point occurs for a launch angle greater than 45°.

2-74 (a) Explain why the maximum range for a man doing a standing broad jump is not obtained for a takeoff angle of 45°. (b) Should the angle be less than or greater than 45°? Explain.

2-75 A frog can jump 0.9 m with a takeoff angle of 45°. (a) What initial velocity does this require? (b) With this same initial velocity directed vertically, how high could a frog jump? (c) The maximum height jumped by frogs is 0.3 m. What are some possible explanations for this difference?

2-76 A flea can jump 0.03 m. (a) If the takeoff angle is 70°, what is the initial velocity? (b) If the flea achieves this velocity in a takeoff distance of 8×10^{-4} m, what is its average acceleration during takeoff?

2-77 A boy can throw a ball a maximum horizontal distance of 60 m. Assuming he can throw equally hard in the vertical direction, how high can he throw a ball?

***2-78** Show that for a ball kicked on level ground, the ratio of the maximum height reached to the range is $\frac{1}{4}\tan\theta_0$.

***2-79** The curvature of the earth becomes important in projectile calculations when the distance traveled R is a significant fraction of the radius of the earth, R_E. Ignoring any possible variations in g, show that (a) the extra time the projectile is in motion is approximately given by $\Delta t \simeq \Delta y/v_{0y} \simeq (R^2/2R_E)/v_{0y}$; (b) the fractional error $\Delta R/R$ is given by $\Delta R/R = v_{0x}^2/gR_E$.

2-80 Using the equation in the preceding problem, find the fractional error in the range of a projectile due to the curvature of the earth if the projectile is fired with a launch angle of 45° and an intended range of 100 km. (The average radius of the earth is 6.38×10^6 m.)

^{c}2-81 (a) Explain why the launch angle θ which gives the maximum range must satisfy $dR/d\theta = 0$. (b) Using this requirement, show that a 45° launch angle produces the maximum range on flat ground.

Additional Reading

Sir James Gray, *How Animals Move*, Cambridge University Press, Cambridge, 1953. Pages 69–80 discuss jumping.

R. McNeill Alexander, *Animal Mechanics*, University of Washington Press, Seattle, 1968. Pages 28–33 discuss jumping.

David F. Griffing, *The Dynamics of Sports—Why That's the Way the Ball Bounces*, Mohican, Loudonville, Ohio, 1982. Basic physics applied to track and field, swimming, water skiing, football, etc.

Scientific American articles:

Stillman Drake and James MacLachlan, Galileo's Discovery of the Parabolic Trajectory, March 1975, p. 102.

Cornelius T. Leondes, Inertial Navigation for Aircraft, March 1970, p. 80.

Graham Hoyle, The Leap of the Grasshopper, January 1958, p. 30.

Miriam Rothschild et al., The Flying Leap of the Flea, November 1973, p. 92.