

EXERCISES

systematic errors	average acceleration
significant figures	instantaneous
translational motion	acceleration
displacement	constant acceleration
average velocity	formulas
uniform motion	gravitational
accelerated motion	acceleration
instantaneous velocity	terminal velocity
slope	mathematical model

REVIEW QUESTIONS

(Answers are given in this chapter just before the supplementary topics.)

Q1-1 The officially recognized set of units for scientific work is the _____.

Q1-2 Experimental data usually contain _____ and _____ errors.

Q1-3 The best way to convert units is to multiply by a factor of _____ for each unit that must be converted.

Q1-4 The change in position is called the _____.

Q1-5 The average velocity is the ratio _____/_____.

Q1-6 The instantaneous velocity is the average velocity evaluated for _____.

Q1-7 The average acceleration is the _____ divided by the _____.

Q1-8 On an $x-t$ graph, the slope equals the _____.

Q1-9 On a $v-t$ graph, the slope equals the _____.

Q1-10 The velocity change equals the area under the _____ graph.

Q1-11 The displacement equals the area under the _____ graph.

Q1-12 In air, a rock falls faster than a feather because of _____.

Q1-13 When an object thrown straight upward reaches its greatest height, its velocity is _____ and its acceleration is _____.

EXERCISES

The student should work out some of the exercises for each section to test his or her general understanding of the concepts before attempting the problems. Answers to most of the odd-numbered exercises

and problems are given at the end of the book, usually to three significant figures. If your answers differ slightly in the last place, this may be due to differences in rounding off intermediate results, rather than to an error in your work. Solutions to half the odd-numbered exercises appear in the Study Guide. Exercises and problems preceded by a ° may require the use of calculus.

Section 1.1 | Measurements, Standards, Units, and Errors

1-1 An acre is 43,560 ft². How large is this in square metres (m²)?

1-2 Convert 40 mi h⁻¹ to metres per second (m s⁻¹).

1-3 A gallon is 231 cubic inches (in.³), and a litre is 1000 cm³. How many litres are there in a gallon?

1-4 A furlong is 220 yards, and a fortnight is 14 days. If a snail moves at 2 m h⁻¹, what is this in furlongs per fortnight?

1-5 A cell membrane is 70 angstrom (Å) units thick. If an angstrom unit is 10⁻¹⁰ m, what is the membrane thickness in (a) metres; (b) micrometres?

1-6 If two quantities have different dimensions, can they be (a) multiplied; (b) added? Give examples to support your answers.

1-7 In the United States land is measured in acres (1 acre = 43,560 ft²). In most other countries it is measured in hectares (1 hectare = 10⁴ m²). How large is a 100-acre farm in hectares?

1-8 The volumes of reservoirs are sometimes measured in acre-feet; that is, a lake with an area of 1 acre and an average depth of 1 ft contains 1 acre-foot of water. If a lake has an area of 100 acres and an average depth of 20 ft, find its volume (a) in acre-feet; (b) in cubic feet; (c) in cubic metres (1 acre = 43,560 ft²).

1-9 Suppose you wanted to know the area of a rectangular room and had a cloth tape measure available to determine its dimensions. What are some random and systematic errors that might affect your result?

1-10 A driver wishes to check the speedometer by traveling at a constant speed along a highway with markers placed every kilometre and having a passenger note the time intervals with a wrist-

watch. Discuss the random and systematic errors involved.

1-11 If a dollar is 7.2 francs and a 0.75-litre bottle of wine costs 20 francs, how much does it cost in dollars per quart?

1-12 A U.S. gallon is 231 in.³. An Imperial gallon, used sometimes in England and in the British Commonwealth, is 277.42 in.³. In Niagara Falls, New York, gasoline is \$1.08 (U.S.) per U.S. gallon. Across the international bridge in Niagara Falls, Ontario, gasoline is \$1.80 (Canadian) per Imperial gallon. If \$1.00 (Canadian) = \$0.72 (U.S.), on which side is gasoline cheaper, and by what percentage?

Section 1.2 | Displacement; Average Velocity

1-13 A car travels 30 km in 45 min on a straight highway. What is its average velocity in kilometres per hour (km h⁻¹)?

1-14 A pilot wishes to fly 2000 km in 4 h. What average velocity in metres per second is required to accomplish this?

1-15 Sketch the position-versus-time graph for a car that starts from rest and is driven 1 km to a store. (Describe the motion in words.)

1-16 A car travels in a straight line at 40 km h⁻¹ for 1 h and at 60 km h⁻¹ for 2 h. (a) How far does it travel? (b) Find the average velocity.

1-17 A woman wants to drive 100 km in 2 h. If she averages 40 km h⁻¹ for the first 1.5 h, what average velocity must she maintain for the remaining time?

1-18 A man runs a 42-km marathon in 2.5 h. Find the average velocity in (a) kilometres per hour (km h⁻¹); (b) metres per second (m s⁻¹).

1-19 A sprinter runs the 100-m dash in 9.8 s. (a) What is the sprinter's average velocity? (b) Since the runner starts from rest, the velocity cannot be constant. Sketch the approximate position-versus-time graph for the runner. Explain the assumptions made.

1-20 Light travels at 3×10^8 m s⁻¹. A light-year is the distance light travels in 1 year, or 365 days. Find the distance in kilometres to the nearest star, which is 4 light-years away from us.

1-21 A falling object moves so that its height x above the ground at time t is given by the equation $x = 100 \text{ m} - (4.9 \text{ s}^{-2})t^2$. Find its average

velocity from (a) $t = 0$ to $t = 2$ s; (b) $t = 2$ to $t = 4$ s.

1-22 A ball thrown straight upward has a height x above the ground which is given by the equation $x = (19.6 \text{ m s}^{-1})t - (4.9 \text{ m s}^{-2})t^2$. Find the average velocity from (a) $t = 0$ to $t = 2$ s; (b) $t = 2$ to $t = 4$ s.

1-23 From the position-versus-time graph of Fig. 1.13, find the average velocity from $t = 0$ s to (a) $t = 10$ s; (b) $t = 20$ s; (c) $t = 40$ s.

1-24 A baseball reaches a batter at 40 m s⁻¹. If home plate is 0.3 m across, how long will the ball be over the plate?

1-25 In 1970, a record was set when a swimmer swam 100 m in 51.9 s. What was his average velocity in km h⁻¹?

Section 1.3 | Instantaneous Velocity

1-26 In a period of 4 h, a man walks 10 km north, turns around, and walks 6 km south. Find his (a) average velocity; (b) average speed.

1-27 John drives 100 km at 100 km h⁻¹ and then drives an additional 100 km at 80 km h⁻¹. Mary makes the same 200-km trip at a constant speed of 90 km h⁻¹. (a) If they leave at the same time, who (if either) will complete the trip first? (b) By how much time will he or she beat the other driver?

1-28 In Fig. 1.13, what is the instantaneous velocity at (a) $t = 5$ s; (b) $t = 15$ s; (c) $t = 25$ s; (d) $t = 35$ s?

1-29 Draw the instantaneous velocity-versus-time graph corresponding to Fig. 1.13.

1-30 Figure 1.14 shows the position of a pendulum versus time. In the interval $t = 0$ to T , when is the velocity (a) zero; (b) positive; (c) negative?

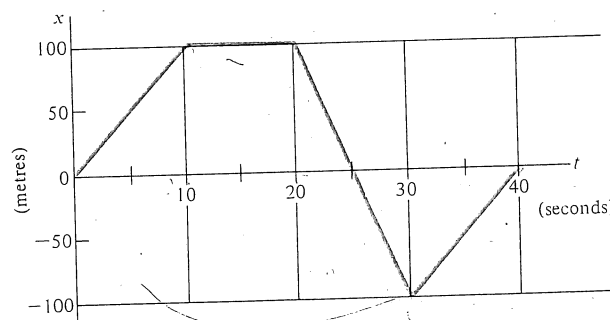


Figure 1.13. Exercises 1-23, 1-28, and 1-29.

EXERCISES

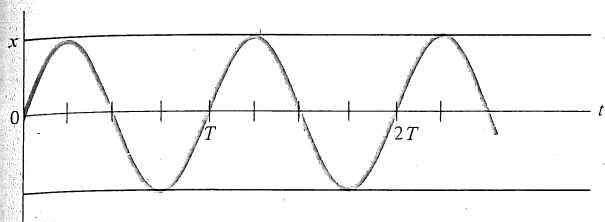


Figure 1.14. The horizontal position of a pendulum versus time. Exercises 1-30, and 1-31, Problem 1-83.

1-31 Figure 1.14 shows the position of a pendulum versus time. In the interval from $t = 0$ to T , when does the velocity have its largest positive and negative values?

1-32 In 1875, Matthew Webb became the first man to swim the English Channel without a life jacket. He required 21 h and 45 min to make the 33.8-km crossing. (a) What was his average velocity in km h^{-1} ? (b) Since he did not swim in a straight line, he actually covered a distance of 60 km. What was his average speed?

1-33 The maximum speed attained by a runner during a 100-m race is reported as 12.5 m s^{-1} . He completed the race in 9.9 s. Are these numbers consistent? Explain.

1-34 The position of an object is given by $x = bt^3$. (a) If x and t are in metres and seconds, respectively, what are the units of b ? (b) What is the instantaneous velocity at time T ? (c) Find the average velocity from $t = 0$ to $t = T$.

1-35 A falling object has a height for $t > 0$ given by $x = (200 \text{ m}) - (4.9 \text{ m s}^{-2})t^2$. (a) Find the instantaneous velocity at $t = 10 \text{ s}$. (b) Find the average velocity from $t = 0 \text{ s}$ to $t = 20 \text{ s}$.

Section 1.4 | Acceleration

1-36 A car proceeds to pass another. Its speed increases from 50 km h^{-1} to 100 km h^{-1} in 4 s. What is the average acceleration?

1-37 A car moves at a constant velocity of 50 m s^{-1} for 20 s. It then slows with a constant acceleration, coming to rest 10 s later. (a) Draw a velocity–time graph for the car. (b) Draw the acceleration–time graph for the car.

1-38 Draw the acceleration–versus–time graph corresponding to Fig. 1.15.

1-39 When does the acceleration corresponding to Fig. 1.15 have (a) a maximum value; (b) a minimum value; (c) a value of zero?

1-40 A car initially moving at 20 m s^{-1} brakes with an acceleration of 2 m s^{-2} . How long will it take for the car to stop?

1-41 A car slowing to a stop has a velocity given by $v = v_0(1 - t/T)$ from $t = 0$ to $t = T$. (a) Find the instantaneous acceleration for $0 < t < T$. (b) Find the average acceleration from 0 to T . (c) Explain the relationship between your answers in parts (a) and (b).

1-42 A falling object has a height for $t > 0$ given by $x = (200 \text{ m}) - (4.9 \text{ m s}^{-2})t^2$. (a) Find the instantaneous acceleration at $t = 10 \text{ s}$. (b) Find the average acceleration from $t = 0 \text{ s}$ to $t = 20 \text{ s}$.

Section 1.5 | Finding the Motion of an Object

1-43 A baseball player hits a pitched ball so that its velocity reverses direction and its speed changes from 30 m s^{-1} to 40 m s^{-1} . The bat moves at an average velocity of 30 m s^{-1} , and it is in contact with the ball for a distance of 0.05 m. (a) For how long are the bat and ball in contact? (b) What is the average acceleration of the ball while it is in contact with the bat?

1-44 A soccer player kicks a ball that is initially at rest on the ground. The player is in contact with the ball for 0.008 s, and the ball leaves his foot at a velocity of 32 m s^{-1} . What was the ball's average acceleration during the kick?

1-45 A spaceship is launched from the moon with an acceleration of 0.1 m s^{-2} . How long will it take the spaceship to achieve a velocity of 10^4 m s^{-1} ?

1-46 An electron in a hydrogen atom has a speed of roughly $3 \times 10^6 \text{ m s}^{-1}$. (a) How long will

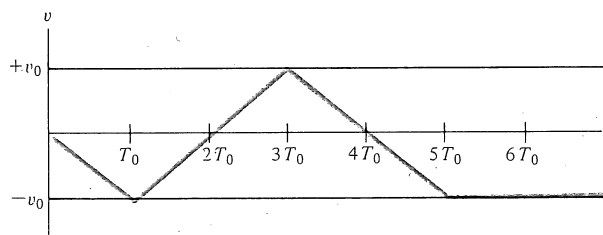


Figure 1.15. Exercises 1-38 and 1-39.

it take the electron to travel a typical atomic distance, 10^{-10} m? (b) If the speed is constant but the velocity reverses direction in that time interval, what is the average acceleration?

1-47 A jet plane accelerates on a runway from rest at 4 m s^{-2} . After 5 s, find its (a) distance traveled; (b) velocity.

1-48 In the accelerator shown in Fig. 1.1, protons emerge with a velocity of $2.5 \times 10^8 \text{ m s}^{-1}$. The accelerator is 0.8 km long. (a) If the acceleration is uniform, how large is it? (b) How long does it take for protons to travel the length of the accelerator?

1-49 A train traveling with a velocity of 30 m s^{-1} stops with a uniform acceleration in 50 s. (a) What is the acceleration of the train? (b) What is the distance traveled before coming to rest?

1-50 A car starts from rest at $t = 0$ and accelerates as shown in Fig. 1.16. Find its velocity at (a) $t = 10$ s; (b) $t = 30$ s. (c) Draw the velocity-versus-time graph.

1-51 Plot the position-versus-time graph corresponding to Fig. 1.17. (Assume the object starts from $x = 0$.)

1-52 Suppose a car can accelerate at 1 m s^{-2} . How large a break in traffic is needed to enter a highway where cars are moving at 20 m s^{-1} if the driver wants to avoid forcing the next car to slow down or to approach closer than 25 m?

1-53 A car moving at 15 m s^{-1} hits a stone wall. (a) A seat-belted passenger comes to rest in 1 m. What average acceleration does this person experience? (b) Another passenger without a seat belt strikes the windshield and comes to rest in

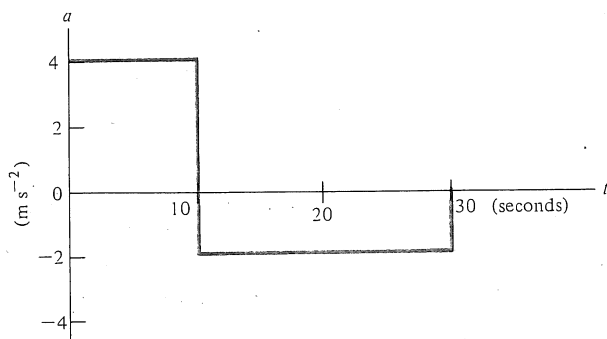


Figure 1.16. Exercise 1-50.

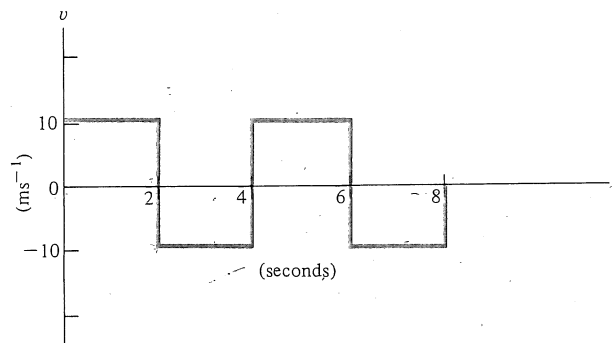


Figure 1.17. Exercise 1-51.

0.01 m. What average acceleration does this person experience?

1-54 A racing car initially at rest has a constant acceleration for 0.5 km. Its speed at the end of this period is 100 m s^{-1} . (a) What is the acceleration of the car? (b) How long does it require to cover the 0.5-km distance?

1-55 A baseball pitcher throws a ball at 40 m s^{-1} . If the acceleration is approximately constant over a distance of 2 m, how large is it?

1-56 A baseball player catches a ball moving at 30 m s^{-1} . (a) If he does not move his hand, the ball comes to rest in his glove over a distance of 1 cm. What is the average acceleration? (b) If he moves his hand as the ball is caught so that it comes to rest over 10 cm, what is the acceleration?

Section 1.6 | The Acceleration of Gravity and Falling Objects

1-57 From what height must water fall to strike a turbine wheel with a vertical downward velocity of 30 m s^{-1} ?

1-58 An antiaircraft shell is fired vertically upward with an initial velocity of 500 m s^{-1} . (a) Compute the maximum height of the shell. (b) How long does it take to reach this height? (c) When will the height be 1000 m?

1-59 A model rocket is fired straight up from ground level with a constant acceleration of 50 m s^{-2} until the engine runs out of fuel after 4 s. Neglecting air resistance, find (a) the height of the rocket when the engine stops; (b) the maximum height reached; (c) the total time duration of the flight.

1-60 A rock falls from a cliff 60 m high. (a) Find the average velocity during the first 3 s of its fall. (b) At what instant of time is the instantaneous velocity equal to the average velocity of part (a)? (c) How long does it take for the rock to fall to the ground?

1-61 A boy standing beside a tall building throws a ball straight up with an initial velocity of 15 m s^{-1} . (a) How high will the ball rise? (b) How long will it take for the ball to reach its maximum height? (c) Another boy reaches out of a window 6 m above the initial position of the ball and attempts to catch the ball. At what times will the ball pass him?

1-62 A rock dropped from the top of a tower strikes the ground in 4 s. (a) Find the velocity of the rock just before it strikes the ground. (b) Find the height of the tower.

1-63 A stone is thrown vertically downward from a bridge with an initial velocity of 10 m s^{-1} . It strikes the water in 3 s. (a) What is the velocity of the stone as it strikes the water? (b) What is the height of the bridge above the water?

1-64 A stone dropped from a bridge strikes the water in 5 s. (a) What is the velocity of the stone when it strikes the water? (b) What is the height of the bridge?

1-65 The hammer of a pile driver strikes the top of a pipe with a velocity of 7 m s^{-1} . From what height did the hammer fall?

1-66 A sandbag dropped from a balloon strikes the ground in 15 s. What was the height of the balloon if it was initially (a) at rest in the air; (b) descending with a velocity of 20 m s^{-1} ?

1-67 A box falls from an elevator that is ascending with a velocity of 2 m s^{-1} . It strikes the bottom of the elevator shaft in 3 s. (a) How long will it take the box to reach its maximum height? (b) How far from the bottom of the shaft was the box when it fell off the elevator? (c) What is the height of the elevator when the box is at its highest point?

1-68 Repeat Example 1-21 with the $+x$ direction downward.

1-69 Repeat Example 1-22 with the $+x$ direction downward.

1-70 A car moving at 30 m s^{-1} (108 km h^{-1}) collides head on with a stone wall. From what height

would the car have to fall to achieve the same results?

1-71 One ball is thrown straight up from a bridge, and another is thrown straight down. Just before they hit the water, which has the larger acceleration? (Neglect air resistance.)

1-72 In 1971, Apollo 15 astronaut David Scott dropped a hammer and a feather on the moon. The feather fell a distance of 0.73 m in the first second. (a) Did the hammer fall a larger or smaller distance in that second? Explain. (b) What is the acceleration due to gravity on the moon?

1-73 A physics student measures her reaction time by having a friend drop a metre stick between her fingers. The metre stick falls 0.3 m before she catches it. (a) What is her reaction time? (b) Estimate the minimum average speed of nerve impulses going from her eye to her brain and then back to her hand.

1-74 A ball is thrown straight up with an initial velocity of 30 m s^{-1} . (a) How far will it travel in the first second after it is released? (b) What is its velocity after 1 s? (c) What is its acceleration after 1 s?

PROBLEMS

The occasional problems preceded by an asterisk are the most difficult. Answers to most of the odd-numbered problems are given in the back of the book. Solutions to half the odd-numbered problems appear in the Study Guide.

1-75 Figure 1.18 shows the position-versus-time graph for an object. Consider the intervals from 0 to T_1 , T_1 to T_2 , and so on. (a) During which time

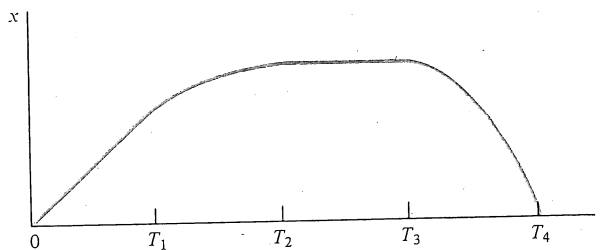


Figure 1.18.

interval is the object at rest? (b) During which time interval is it moving with constant velocity? (c) During which time interval does it have a positive velocity? (d) During which time interval does it have a negative velocity? (e) During which time interval does it have a positive acceleration? (f) During which time interval does it have a negative acceleration?

1-76 A car traveling at 20 m s^{-1} hits a stone wall. The driver, who is wearing a shoulder harness and seat belt, moves forward 1.0 m as the car stops. Assuming her acceleration is uniform, find (a) her average velocity during the collision; (b) her acceleration.

1-77 We saw that when you throw an object upward, in the absence of air resistance, it takes as long for the object to come down again as to reach its highest point. Are these times equal when there is air resistance? Explain.

1-78 Galileo concluded that falling objects have a constant acceleration because their distance varies with the square of the elapsed time: $x = ct^2$. Is this equation valid when air resistance is present? If not, explain qualitatively how the equation would have to be modified.

1-79 At the instant a bowler releases the ball, her hand is moving horizontally with a speed of 6 m s^{-1} relative to her body. If she is moving forward at a speed of 1 m s^{-1} , what is the velocity of the ball with respect to the floor?

1-80 When a bowling ball is released, a bowler's hand is moving relative to his forearm at 0.82 m s^{-1} . The forearm is moving relative to the upper arm at 0.55 m s^{-1} , and the upper arm is moving at 5.26 m s^{-1} relative to the shoulder. (The velocities refer to the outermost ends of the respective parts of the body and are all horizontal.) If the bowler's shoulder is moving relative to the floor at 1.43 m s^{-1} , how fast is the ball thrown?

1-81 A girl rows 12 km downstream in 2 h . Her return trip takes 3 h . (a) How fast can she row in still water? (b) How fast is the current?

1-82 A boy is on a train moving at 70 km h^{-1} . Relative to the ground, how fast is he moving if

he runs at 15 km h^{-1} (a) toward the front of the train; (b) toward the rear?

1-83 The position-versus-time graph for a pendulum is shown in Fig. 1.14. Sketch the velocity-versus-time and acceleration-versus-time graphs for the motion.

1-84 A sled starting from rest slides down a hill with uniform acceleration. It travels 12 m in the first 4 s . When will the sled have a velocity of 4 m s^{-1} ?

1-85 A racing car initially at rest accelerates for one-quarter of a kilometre and then brakes to a stop in an additional one-half kilometre. (a) Sketch an approximate velocity-versus-time graph. (b) Sketch an acceleration-versus-time graph.

1-86 Plane A, flying at 500 m s^{-1} , is $10,000 \text{ m}$ directly behind plane B, moving in the same direction at 400 m s^{-1} . The pilot of plane A fires a missile that accelerates at 100 m s^{-2} . How long will it take for the missile to reach plane B? (Neglect the effects of the gravitational acceleration.)

1-87 A dog running at 10 m s^{-1} is 30 m behind a rabbit moving at 5 m s^{-1} . When will the dog catch up with the rabbit? (Both velocities remain constant.)

1-88 A rocket-powered experimental sled carrying a test pilot is brought to rest from 200 m s^{-1} over a distance d . If the pilot is not to be subjected to an acceleration greater than six times that of gravity, what is the minimum value of d ?

***1-89** The world record for the 100-m dash is 9.95 s , and for the 60-m dash it is 6.45 s . Assume a sprinter accelerates at a constant rate up to a maximum velocity that is maintained for the remainder of the race, no matter how long it is. (a) Find the acceleration. (b) What is the duration of the acceleration period? (c) What is the maximum velocity? (d) The record for the 200-m dash is 19.83 s , while for the 1000-m run it is about 133.9 s . Are these times consistent with the assumptions made?

1-90 A sandbag is dropped from an ascending balloon that is 300 m above the ground and is

ANSWERS TO REVIEW QUESTIONS

ascending at 10 m s^{-1} . (a) What is the maximum height of the sandbag? (b) Find the position and velocity of the sandbag after 5 s. (c) How long does it take for the sandbag to reach the ground from the time it was dropped?

***1-91** In the television show "The Six Million Dollar Man," Colonel Austin had superhuman capabilities. In one episode he tries to catch a man fleeing in a sports car. The distance between them is 100 m when the car begins to accelerate with a constant acceleration of 5 m s^{-2} . Colonel Austin runs at a constant speed of 30 m s^{-1} . Show that he cannot catch the car, and find his distance of closest approach.

1-92 In Fig. 1.8b, the shaded area has a triangular portion (above the v_0 line) and a rectangular portion. Show that the sum of their areas reduces to the right-hand side of Eq. 1.8.

1-93 (a) A magazine article states that cheetahs are the fastest sprinters in the animal world and that a cheetah was observed to accelerate from rest to 70 km h^{-1} in 2 s. What average acceleration in m s^{-2} does this require? (b) The article also says the cheetah covered 60 m during that 2-s interval. How large a constant acceleration is implied by this statement? Does it agree with your result in (a)? (c) Accelerations substantially greater than g are difficult for an animal or an automobile to attain, because there is a tendency to slip even on very rough ground with larger acceleration. Given this information, can you guess which number is wrong in the article?

1-94 A rock is dropped into a well and a splash is heard 3 s later. If sound travels in air at 344 m s^{-1} , how deep is the well?

1-95 A lightning flash is seen, and 5 s later thunder is heard. Assuming they are produced simultaneously, how far away is the flash? (In air, sound travels at 344 m s^{-1} and light at $3.00 \times 10^8 \text{ m s}^{-1}$.)

1-96 An estimate of the difficulties involved in space exploration beyond the solar system can be seen from the following calculation. (a) The distance to the moon from the earth is $3.84 \times 10^8 \text{ m}$. Present-day spaceships require about 24 hours to

reach the moon. What is the average velocity of these spaceships? (b) The nearest star to our solar system is about 4 light-years away, where a light-year is the distance traveled by light in 1 year (365 days) and the speed of light is $3.00 \times 10^8 \text{ m s}^{-1}$. How many years would it take to reach this star if the spaceship had the velocity calculated in (a)?

***1-97** A mass on the end of a spring moves so that its position is given by $x = A \sin(2\pi t/T)$. (a) Show that the motion repeats after a time T . (T is called the *period*.) (b) What is the greatest distance the mass moves from $x = 0$? (c) Find the instantaneous velocity as a function of time. (d) Find the instantaneous acceleration as a function of time. (e) Show that a is proportional to $-x$ and find the constant of proportionality.

***1-98** An object falling through a fluid has a velocity at a time t given by $v = v_f(1 - e^{-t/T})$. ($e = 2.718 \dots$ is the base of natural logarithms. See Appendix B.10.) (a) Find the velocities at $t = 0$, T , and ∞ . (b) Explain why v_f is called the terminal velocity. (c) Show that the instantaneous acceleration is $(v_f - v)/T$. (d) Give a qualitative explanation of your results.

***1-99** The position of an object moving along the x axis is given by $x = A + Bt^2 - Ct^3$, where A , B , and C are positive numbers. At time t , find (a) the velocity; (b) the acceleration. (c) At what time is the value of x greatest?

***1-100** A swimmer can swim at a velocity v in still water. She swims upstream a distance d against the current, which has a velocity u . She then swims back to her starting point. (a) How long does it take her to make the round-trip? (b) What is her average speed for the trip? (c) For what value of u is her average speed the greatest?

ANSWERS TO REVIEW QUESTIONS

Q1-1, Système Internationale (S.I.); **Q1-2**, random, systematic; **Q1-3**, one; **Q1-4**, displacement; **Q1-5**, displacement, elapsed time; **Q1-6**, an extremely short time; **Q1-7**, velocity change, elapsed time; **Q1-8**, instantaneous velocity; **Q1-9**, instantaneous acceleration; **Q1-10**, $a-t$; **Q1-11**, $v-t$; **Q1-12**, air resistance; **Q1-13**, 0, g (downward).

If v is constant, it can be taken out of the integral. The equation then becomes $x_2 - x_1 = v(t_2 - t_1)$, or $\Delta x = v \Delta t$, as expected.

We illustrate the use of these results with an example of an accelerating car.

Example 1.24

A car starts from rest at time $t = 0$. It moves with an acceleration which diminishes linearly to zero according to the formula $a = a_0(1 - t/T)$, where $a_0 = 2 \text{ m s}^{-2}$ and $T = 10 \text{ s}$ (Fig. 1.22a). (a) What is the velocity of the car when the acceleration reaches zero at $t = T$? (b) How far does the car move during the acceleration period? (c) What is the average velocity from $t = 0$ to $t = T$?

(a) The velocity at time $t = 0$ is zero. At a later time t , the velocity is the integral of the acceleration from 0 to t . Since we want the velocity at time t , we use t' as the integration variable. Thus

$$v = \int_0^t a_0 \left(1 - \frac{t'}{T}\right) dt' = a_0 \int_0^t \left(1 - \frac{t'}{T}\right) dt'$$

Now according to Eq. B.38 in Appendix B.12, $\int t'^n dt' = t'^{n+1}/(n+1)$. Thus $\int dt' = t'$, $\int t' dt' = t'^2/2$, and v is given by

$$v = a_0 \left[t' - \frac{t'^2}{2T} \right]_0^t = a_0 \left(t - \frac{t^2}{2T} \right)$$

This is the velocity at any time t during the acceleration period (Fig. 1.22b). Substituting $t = T$ into this result, the velocity at the end of the acceleration period is

$$v = a_0 \left(T - \frac{T^2}{2T} \right) = \frac{a_0 T}{2}$$

Using $a_0 = 2 \text{ m s}^{-2}$ and $T = 10 \text{ s}$, we have

$$v = \frac{(2 \text{ m s}^{-2})(10 \text{ s})}{2} = 10 \text{ m s}^{-1}$$

(b) We find the position change by integrating the velocity. If we measure x from the starting point, then $x = 0$ at $t = 0$, and at later times

$$x = \int_0^t a_0 \left(t' - \frac{t'^2}{2T} \right) dt'$$

Using Eq. B.38, we have $\int t' dt' = t'^2/2$, $\int t'^2 dt' = t'^3/3$, and

$$x = a_0 \left[\frac{t'^2}{2} - \frac{t'^3}{6T} \right]_0^t = a_0 \left(\frac{t^2}{2} - \frac{t^3}{6T} \right)$$

This is the distance the car has traveled up to time t (Fig. 1.22c). At $t = T$,

$$\begin{aligned} x &= a_0 \left(\frac{T^2}{2} - \frac{T^3}{6T} \right) = \frac{a_0 T^2}{3} \\ &= \frac{(2 \text{ m s}^{-2})(10 \text{ s})^2}{3} = 66.7 \text{ m} \end{aligned}$$

The car traveled 66.7 m while accelerating.

(c) The average velocity is the displacement divided by the elapsed time,

$$\bar{v} = \Delta x / \Delta t = (66.7 \text{ m}) / (10 \text{ s}) = 6.67 \text{ m s}^{-1}$$

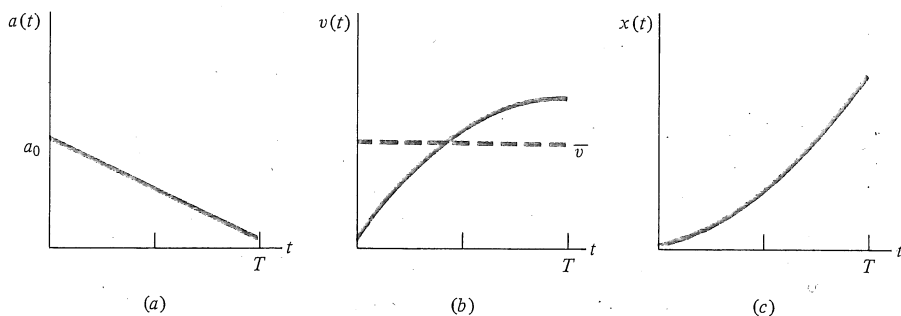
Note that \bar{v} is *not* equal to the average of the initial and final velocities, $\frac{1}{2}(0 + a_0 T/2) = \frac{1}{2}(10 \text{ m s}^{-1}) = 5 \text{ m s}^{-1}$ (Fig. 1.22b). The average velocity is usually different from the average of the initial and final velocities when the acceleration is not constant.

EXERCISES ON SUPPLEMENTARY TOPICS

Section 1.8 | Vertical Jumping

1-101 A salmon jumps vertically out of the water at an initial velocity of 6 m s^{-1} . (a) How high will it jump? (b) How long will the salmon be out of the water?

Fig. 1.22. Example 1.24 (a) The acceleration of a car satisfies $a = a_0(1 - t/T)$. (b) The velocity of the car versus time if it starts from rest at $t = 0$. Note the average velocity is more than half the final velocity. (c) Its position versus time.



1-102 How high will a woman jump if her take-off velocity is the same as that of the flea?

1-103 From Table 1.4, compute the average takeoff acceleration and takeoff velocity for a locust, assuming the acceleration is constant. Compare these results with that for the human of Example 1.23.

1-104 An astronaut wearing a space suit can jump 0.5 m vertically at the surface of the earth. The gravitational acceleration on Mars is 0.4 times that on the earth. If his takeoff velocity is the same, how high can the astronaut jump on Mars?

1-105 If a human could achieve a takeoff acceleration equal to that of the flea, how high could she jump? (Assume the acceleration distance is still 0.5 m.)

1-106 Compute the takeoff times for the human, bushbaby, and flea of Table 1.4.

Section 1.9 | Finding the Motion Using Integration

1-107 The velocity of an object is $v = bt^2$. If the object is at the origin at $t = 0$, where is it at $t = T$?

1-108 An object has an acceleration $a = kt$, where $k = 3 \text{ m s}^{-3}$. At $t = 0 \text{ s}$, it is at the origin and has a velocity $v_0 = 10 \text{ m s}^{-1}$. (a) What is the velocity at $t = 10 \text{ s}$? (b) What is the position at this time?

PROBLEMS ON SUPPLEMENTARY TOPICS

1-109 The velocity of a small object starting from rest at $t = 0$ and falling through a fluid is found to obey the formula $v = v_f(1 - e^{-bt})$, where b is a constant. ($e = 2.718$. . . is the base of the natural logarithms. See Appendix B.10.) Find the position at time $t > 0$ if the object is at the origin at $t = 0$.

1-110 Using integration techniques, derive the uniform acceleration formulas for v and for Δx , Eqs. 1.5 and 1.8.

1-111 A protein molecule in water is effectively weightless because the buoyancy of the water balances the gravitational attraction. If the molecule is set in motion, it eventually comes to rest because of the resistance of the fluid to its motion. The acceleration is opposite to the velocity

and proportional to its magnitude, $a = -cv$. Find the subsequent velocity if the initial velocity at $t = 0$ is v_0 . [Hint: Find dv/v and integrate.]

1-112 We defined the average velocity \bar{v} as the displacement Δx divided by the elapsed time Δt . Show that \bar{v} is also equal to the *time-averaged* instantaneous velocity:

$$\bar{v} = \frac{\int_{t_1}^{t_2} v \, dt}{\int_{t_1}^{t_2} dt}$$

1-113 The preceding problem notes that the average velocity \bar{v} equals the time-averaged instantaneous velocity. A *position-averaged* velocity \bar{v}' can be defined by

$$\bar{v}' = \frac{\int_{x_1}^{x_2} v \, dx}{\int_{x_1}^{x_2} dx}$$

Except in special cases, \bar{v} and \bar{v}' are not equal. Consider an object dropped from $x_1 = 0$ at $t_1 = 0$. Then $a = g$, and $v = gt = (2gx)^{1/2}$. Show that with $v_2 = at_2$, (a) $\bar{v} = v_2/2$; (b) $\bar{v}' = 2v_2/3$.

1-114 Two observers study the motion of an object along the ground. The first observer is at rest relative to the ground and observes the object to have a velocity $v_1(t)$ at time t . The second observer is moving at a constant velocity u away from the first and observes the velocity of the object to be $v_2(t)$. (a) How are v_1 and v_2 related? (b) Find $x_1(t) - x_2(t)$, assuming that both are zero at time $t = 0$. (Do not assume that v_1 and v_2 are constant.) (c) Find $a_1(t)$ and $a_2(t)$.

1-115 An object moving at the end of a spring has an acceleration $a = A \cos \omega t$, where A and ω are constants. For a full oscillation, the position of the object averages out to $x = 0$. At time $t = 0$ its velocity is zero. (a) Find the velocity v at time t . (b) Find the position at time t .

Additional Reading

Alfred M. Bork and Arnold B. Arons, Resource Letter Col R-1 on Collateral Reading in Physics, *American Journal of Physics*, vol. 35, 1967, p. 1. A bibliography. Marjorie Nicholson, Science and Literature, *American Journal of Physics*, vol. 33, 1965, p. 175. A bibliography.