

# Lethbridge Number Theory and Combinatorics Seminar



Monday — January 20, 2014

Room: **B650** ← *Note change of room!*

Time: 12:00 to 12:50 p.m.

**Eric Naslund**  
(Princeton University)

## A density increment approach to Roth's theorem in the primes

*Abstract:* By combining Green and Taos transference principle with a density increment argument, we show if  $A$  is a set of prime numbers satisfying

$$\sum_{a \in A} \frac{1}{a} = \infty,$$

then  $A$  must contain a 3 term arithmetic progression. The previous methods of Helfgott and De Roton, and the speaker, used the transference principle to move from a set of primes to a dense subset of integers by considering the  $L^2$  and  $L^p$ -norms of a smoothed version of the indicator function of  $A$ . Instead, we work directly with the  $L^\infty$ -norm, and exploit the structure of  $A$  to obtain increased density on a large subprogression when  $A$  contains no arithmetic progressions. By iterating we show that for any constant  $B > 0$ , if  $A$  is a subset of primes contained in  $\{1, \dots, N\}$  with size at least

$$|A| \gg_B \frac{N}{(\log N)(\log \log N)^B},$$

then  $A$  contains a three term arithmetic progression. By combining Green and Taos transference principle with a density increment argument, we show if  $A$  is a set of prime numbers satisfying

$$\sum_{a \in A} \frac{1}{a} = \infty,$$

then  $A$  must contain a 3 term arithmetic progression. The previous methods of Helfgott and De Roton, and the speaker, used the transference principle to move from a set of primes to a dense subset of integers by considering the  $L^2$  and  $L^p$ -norms of a smoothed version of the indicator function of  $A$ . Instead, we work directly with the  $L^\infty$ -norm, and exploit the structure of  $A$  to obtain increased density on a large subprogression when  $A$  contains no arithmetic progressions. By iterating we show that for any constant  $B > 0$ , if  $A$  is a subset of primes contained in  $\{1, \dots, N\}$  with size at least

$$|A| \gg_B \frac{N}{(\log N)(\log \log N)^B},$$

then  $A$  contains a three term arithmetic progression.

**EVERYONE IS WELCOME!**

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